If eqn of parabola is $y^2 = 4ax$ then

1) Co-ordinates of focus are $(a, 0)$ and the eqn of directrix is $x + a = 0$

2) Change of origin to 'S' without altering the direction of the axes then the equation of parabola becomes $y^2 = 4a(x+a)$

3) By change of origin to 'A' without altering the direction of the axes then the equation of parabola becomes $y^2 = 4a(x-a)$.

4) The curve passes through origin $(0,0)$ and is symmetrical w.r.t. x-axis and for every value of x there are two values of 'y' with equal magnitude but opposite sign.

5) 'x' cannot be negative as y^2 is always positive. Therefore the curve lies wholely on the positive side of y-axis.

Axis – The line through the focus $\perp$ the directrix is called the axis of parabola

Latus Rectum – The double ordinate LSL_1 passing through the focus and $\perp$ to the axis of the parabola is called latus rectum.

i.e., latus ~~of~~ rectum is $LS = 2a$.

and length of latus rectum $= 4a$.

Focal radius - SP is called the focal radius to the point 'P' on the curve and
$SP = PM = AN = x + a$

Focal Chord - Any chord passing through the focus is called a focal chord.

Equation of the chord joining the points t_1 and t_2 on the parabola $y^2 = 4ax$ is
$y(t_1 + t_2) = 2x + 2at_1t_2$.
If the chord passes through focus $(a, 0)$
if $t_1 t_2 = -1$.

Ques) Find the equation to the parabola whose focus is $(2,3)$ and directrix is $x - 2y - 6 = 0$.

Soln The eqn of parabola by definition is

$$(x-\alpha)^2 + (y-\beta)^2 = \frac{(ax+by+c)^2}{a^2+b^2}$$

$$\Rightarrow (x-2)^2 + (y-3)^2 = \frac{(x-2y-6)^2}{1^2+(-2)^2}$$

$$\Rightarrow 5(x^2 + 4 - 4x + y^2 - 6y + 9) = x^2 + 4y^2 + 36 - 4xy - 12x + 24y$$

$$\Rightarrow 5x^2 + 20 - 20x + 5y^2 - 30y + 45 = x^2 + 4y^2 + 36 - 4xy - 12x + 24y$$

$$\Rightarrow 5x^2 - x^2 + 5y^2 - 4y^2 + 4xy = 20x - 12x + 30y + 24y + 36 - 45$$

$$\Rightarrow (2x+y)^2 = 54y - 2x - 29$$

Ans

Ques If the point (2,3) is the focus and $x = 2y + 6$ is the directrix of a parabola then find:-

(i) the equation of the axis
(ii) the co-ordinates of the vertex
(iii) length of the latus rectum
(iv) Equation of the latus rectum.

Soln :(i) Since the axis of parabola is the line passing through the focus ⊥ to the directrix so the eqn of line passing through (2,3) is

$$y - 3 = m(x-2)$$
$$\Rightarrow mx - y = 3 - 2m$$

∵ the directrix is $x - 2y = 6$

∴ $m \times \frac{1}{2} = -1 \Rightarrow m = -2$.

∴ Required eqn of axis is

$$+2x + y = 3 - 2(-2)$$
$$\cancel{\Rightarrow y - 3 = 2x + 6}$$
$$\Rightarrow 2x + y = (7)$$

(ii) Co-ordinates of point of intersection of directrix $x = 2y + 6$ and axis $2x + y = 7$ are obtained by solving the equations that is $(4, -1)$.

∴ The vertex is middle point of points $(4,-1)$ and focus $(2,3)$. Therefore co-ordinates of vertex are $\left(\frac{4+2}{2}, \frac{3-1}{2}\right)$

$$= (3, 1)$$

(iii) Since $a = \sqrt{(3-2)^2 + (1-3)^2} = \sqrt{5}$

$\therefore$ the length of latus rectum $= 4a$

$= 4\sqrt{5}$

(iv) $\because$ the latus rectum is the line passing through the focus and parallel to directrix

$\therefore$ the eqn is $x - 2y + C = 0$

where C is given by $2 - 2(3) + C = 0$

$\Rightarrow C = 4$.

$\Rightarrow x - 2y + 4 = 0$ is required eqn.